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10th Physics

Unit 18 Radioactivity

Numerical Problems

1. Half life = $T_{1/2}$
2. Total quantity = N_0
3. left quantity = N
4. Number of half-lives = n
5. Total elapsed time = Δt
   (Total passed time)

Important Formulas

1. $N = \left(\frac{1}{2}\right)^n \times N_0 \Rightarrow N = \frac{1}{2^n} \times N_0$ (New Book)

   $\Rightarrow N = \frac{1}{2^t} \times N_0$ (old Book)

2. Total elapsed time $\Delta t = n \times T_{1/2}$

   = Number of Half lives × Half life

3. Mathematics rule:

   (i) $2^2 = 2^n$

   $\Rightarrow$

   $\boxed{n = 2}$

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1. A radioactive Substance has a half-life of eight months. In how much time, three-fourth of the Substance will decay?

Data:

Half-life $T_{1/2}$ = 8 months
Total elapsed time Δt = ?
Quantity decayed = $\frac{3}{4} N_0$

Total quantity = N_0

Quantity left $N = N_0 - \frac{3}{4} N_0$ $\quad \because 1 - \frac{3}{4}$

$= \frac{1}{4} N_0$ $\quad = \frac{4-3}{4}$

$= \frac{1}{4}$

Solution:

Total elapsed time $\Delta t = n \times T_{1/2}$ ——①

(Number of Half lives ↑, Half life ↗)

→ To Find number of half-lives 'n'

$$N = N_0 \times \left(\frac{1}{2}\right)^n$$

$$N = N_0 \times \frac{1}{2^n}$$

Putting value of N

$$\frac{1}{4} N_0 = N_0 \times \frac{1}{2^n}$$

$$\frac{1}{4} = \frac{1}{2^n}$$

$$\frac{1}{2^2} = \frac{1}{2^n}$$

$$2^2 = 2^n$$

$\Rightarrow$

$$\boxed{n = 2}$$

Now From ①

Total elapsed time $\Delta t = n \times T_{1/2}$

$= 2 \times 8$ months

$$\boxed{\Delta t = 16 \text{ months}}$$

2. In 420 days, the activity of a Sample of polonium (Po) to one eight of its initial value. What is the half-life of polonium?

Data:

Total elapsed time $\Delta t = 420$ days

Quantity left $N = \frac{1}{8} N_0$

Half-life $T_{1/2} = ?$

$\because \Delta t = n \times T_{1/2}$

$\frac{\Delta t}{n} = T_{1/2}$

To find number of half-lives "n"

$$N = N_0 \times \frac{1}{2^n}$$

$$\frac{1}{8} N_0 = N_0 \times \frac{1}{2^n}$$

$$\frac{1}{8} = \frac{1}{2^n}$$

$$\frac{1}{2^3} = \frac{1}{2^n}$$

$$\Rightarrow \quad 2^3 = 2^n$$

$$\Rightarrow \quad \boxed{n = 3}$$

2	8
2	4
2	2
	1

Now

$$\Delta t = n \times T_{1/2}$$

$$\frac{\Delta t}{n} = T_{1/2}$$

$$\Rightarrow \quad T_{1/2} = \frac{\Delta t}{n}$$

$$= \frac{420}{3}$$

$$\boxed{T_{1/2} = 140 \text{ days}}$$

3. The half-life of radium is about 1600 years. In how much time, 25 g will remain un-decayed from original 100 g?

Data:

Half-life $T_{1/2} = 1600$ years

Total elapsed time $\Delta t = ?$

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Quantity left $N = 25g$

Total quantity $N_0 = 100g$

Solution:

$$\Delta t = n \times T_{1/2}$$

→ To find number of half-lives "n"

$$N = N_0 \times \frac{1}{2^n}$$

$$25 = 100 \times \frac{1}{2^n}$$

$$\frac{25}{100} = \frac{1}{2^n}$$

$$\frac{1}{4} = \frac{1}{2^n}$$

$$\frac{1}{2^2} = \frac{1}{2^n}$$

$$2^2 = 2^n$$

$\Rightarrow$

$$\boxed{n = 2}$$

Now

$$\Delta t = n \times T_{1/2}$$

$$= 2 \times 1600$$

$$\boxed{\Delta t = 3200 \text{ years}}$$

4. One eight of the initial mass of a Certain radioactive isotope remain undecayed after one hour. What is the half-life of isotope in minutes?

Data:

Quantity left $N = \frac{1}{8} N_0$

Total elapsed time $\Delta t = 1 \text{ hour} = 60 \text{ min}$

Half-life $T_{1/2} = ?$

Solution:

$$\Delta t = n \times T_{1/2}$$

$$\frac{\Delta t}{n} = T_{1/2}$$

To Find Number of half-lives "n"

$$N = N_0 \times \frac{1}{2^n}$$

$$\frac{1}{8} N_0 = N_0 \times \frac{1}{2^n}$$

$$\frac{1}{8} = \frac{1}{2^n}$$

$$\frac{1}{2^3} = \frac{1}{2^n}$$

$$2^3 = 2^n$$

$\Rightarrow$

$$\boxed{n = 3}$$

Now

$$\Delta t = n \times T_{1/2}$$

$$\frac{\Delta t}{n} = T_{1/2}$$

$$\Rightarrow \quad T_{1/2} = \frac{\Delta t}{n}$$

$$= \frac{60}{3}$$

$$\boxed{T_{1/2} = 20 \text{ minutes}}$$

5. What proportion of a radium Sample would be decayed after 8000 years? (Half-life of radium is 1600 years).

Data:

Sample decayed = ?

Total elapsed time Δt = 8000 years

Half-life $T_{1/2}$ = 1600 year

Solution:

Sample decayed = Total quantity - Quantity left —①

Let Total quantity = N_0

Quantity left = $N = N_0 \times \frac{1}{2^n}$ —— ②

To find number of Half lives "n"

$$\Delta t = n \times T_{1/2}$$

$$\frac{\Delta t}{T_{1/2}} = n$$

$$\Rightarrow \quad n = \frac{\Delta t}{T_{1/2}}$$

$$= \frac{8000}{1600}$$

$$\boxed{n = 5}$$

Now from ②

$$\text{Quantity left } N = N_0 \times \frac{1}{2^n}$$

$$= N_0 \times \frac{1}{2^5}$$

$$= N_0 \times \frac{1}{32}$$

Suppose
$N_0 = 100g$
$= 100 \times \frac{1}{32}$
$= 3.125$

Now from ①

$$\text{Quantity decayed} = \text{Total quantity} - \text{Quantity left}$$

$$= N_0 - N_0 \times \frac{1}{32}$$

$= 100 - 3.125$
$= 96.875$

$$= N_0 \left(1 - \frac{1}{32}\right)$$

$$= N_0 \left(\frac{32-1}{32}\right)$$

$$= N_0 \left(\frac{31}{32}\right)$$

$$\boxed{\text{Quantity decayed} = \frac{31}{32} N_0}$$

$= \frac{31}{32} \times 100$
$= 96.875$

6. The half-life of a radioactive element is 15 days. If we have initially 50000 atoms of this radioactive atom then how many numbers of atoms are left over after 45 days?

Data:

Half-life $T_{1/2} = 15$ days

Total quantity $N_0 = 50000$ atoms

Quantity left $N = ?$

Total elapsed time $\Delta t = 45$ days

Solution:

$$\text{Quantity left } N = N_0 \times \frac{1}{2^n} \quad \text{——} ①$$

Number of half-lives "n"

$$n = \frac{\Delta t}{T_{1/2}} \qquad \because \Delta t = n \times T_{1/2}$$

$$= \frac{45}{15}$$

$$\boxed{n = 3}$$

Now From ①

$$N = N_0 \times \frac{1}{2^n}$$

$$= 50,000 \times \frac{1}{2^3}$$

$$= 50,000 \times \frac{1}{8}$$

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$$N = \frac{50,000}{8}$$

$$\boxed{N = 6250 \text{ atoms}}$$

7. What percentage of the original quantity of a radioactive material is left after five half lives approximately?

Data:

Percentage of quantity left = N % = ?

Number of half-lives $n = 5$

Total quantity $= N_0$

Solution:

Quantity left $N = N_0 \times \frac{1}{2^n}$

$$N = N_0 \times \frac{1}{2^5}$$

$$N = N_0 \times \frac{1}{32}$$

$$\frac{N}{N_0} = \frac{1}{32}$$

$$\% \frac{N}{N_0} = \frac{1}{32} \times 100$$

$$\boxed{\% \frac{N}{N_0} = 3.125\ \%}$$

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OR

Percentage of quantity left $= \frac{N}{N_0} \times 100$

$$= \frac{\frac{N_0}{32}}{N_0} \times 100$$

$$= \left(\frac{N_0}{32} \div N_0 \right) \times 100$$

$$= \left(\frac{N_0}{32} \times \frac{1}{N_0} \right) \times 100$$

$$= \frac{1}{32} \times 100$$

$$= 3.125\ \%$$